

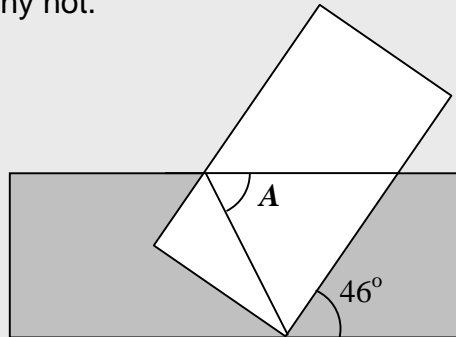
Examples 2 in Basic Geometry

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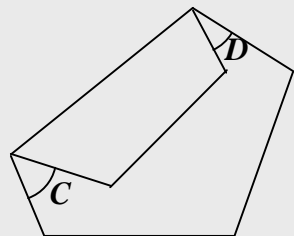
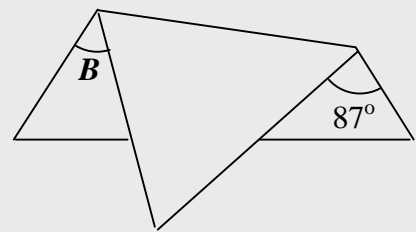
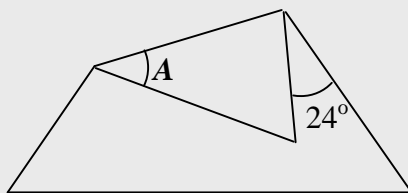
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Examples 2

0. Assuming two rectangles are placed the way below, find the angle A . If it cannot be found, explain why not.



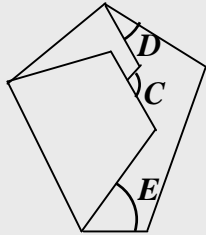
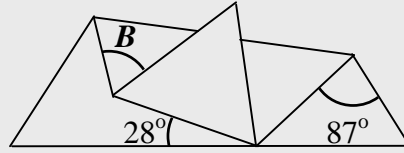
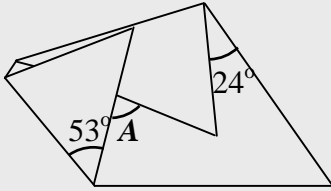
1. Assuming two regular (equilateral) triangles and a regular pentagon are folded the way below, find the three angles A , B , and C .



$D = 23^\circ$.

2

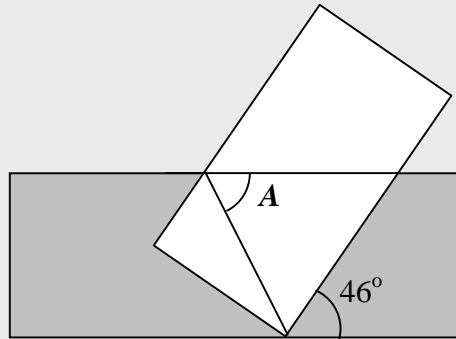
2. Assuming two regular (equilateral) triangles and a regular pentagon are folded the way below, find the three angles A , B , and C .



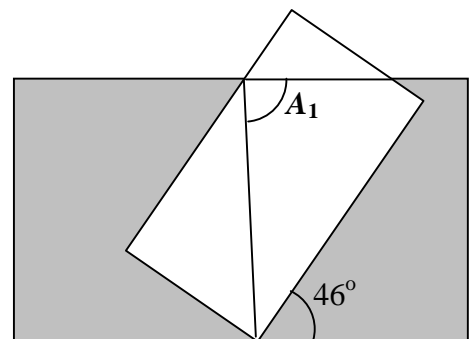
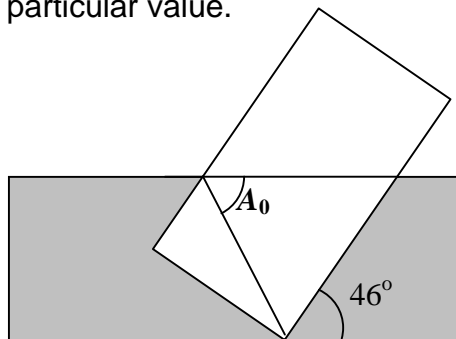
$D = 23^\circ$, and $E = 45^\circ$.

Suggestions or Solutions To the Problem 0

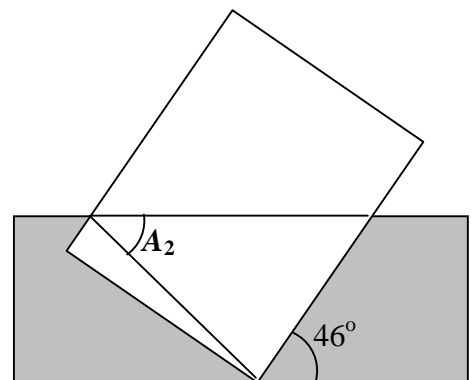
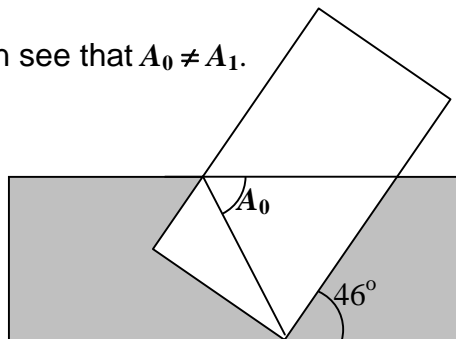
Assuming two rectangles are placed the way below, find the angle A . If it cannot be found, explain why not.



We cannot find the angle A . That's because the rectangles are not specific. In other words, the rectangles can have any dimensions, that is, they can have any lengths or widths. So the angle A cannot stay the same, and in turn, cannot have a particular value.



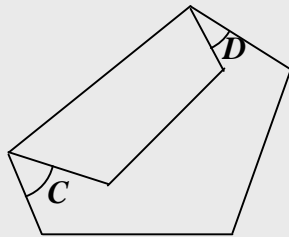
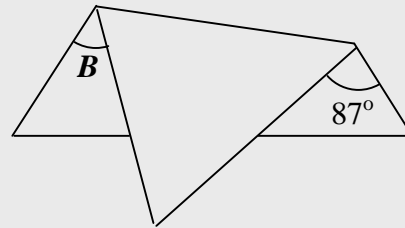
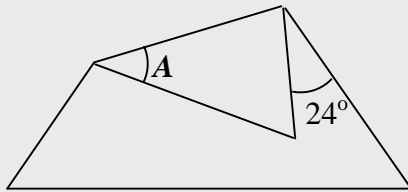
We can see that $A_0 \neq A_1$.



We can see that $A_0 \neq A_2$.

Suggestions or Solutions To the Problem 1

Assuming two regular (equilateral) triangles and a regular pentagon are folded the way below, find the three angles A , B , and C .

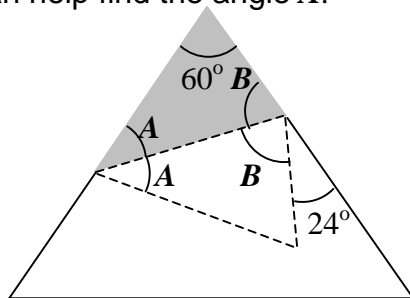


$$D = 23^\circ.$$

Unfolding the figures above, we can unfold the way to the solution.

So first, unfolding one of the three figures the way below, we can see some angles that can help find the angle A .

Fig. 1.0



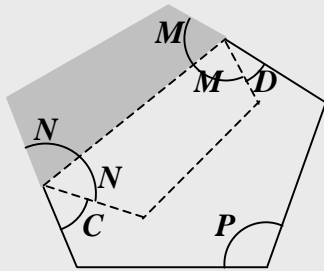
First, we can see two identical triangles.

One is in gray, and the other is made of dashed line segments.

Next, we know the figure unfolded is a regular triangle. So each angle in it is 60° .

Next, we have $B + B + 24^\circ = 180^\circ$. So we get $2B = 180^\circ - 24^\circ = 156^\circ \Rightarrow B = 78^\circ$.

Fig. 1.2



$$D = 23^\circ.$$

Then first, we can see two identical tetragons. One is in gray, and the other is made of dashed line segments. So we can indicate two pairs of same angles the way above.

Thus, we get $M + M + D = 180^\circ$. And we have $D = 23^\circ$, too.

So we get $2M = 180^\circ - D = 180^\circ - 23^\circ = 157^\circ \Rightarrow M = 78.5^\circ$.

Next, we have $N + N + C = 180^\circ$. So finding N , we can get C . How can we find N though?

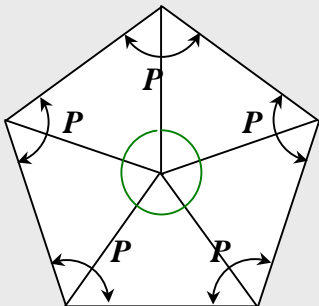
We know that the figure unfolded is a regular pentagon.

So every vertex angle in it is the same, and is P as shown in the figure above.

What then, is the angle P ?

We can partition a regular pentagon into five triangles the way below.

Fig. 1.3



Then first, we assumed that in a regular pentagon, every vertex angle is P .
 And next, we can see that at the center, the sum of all the five angles is 360° .
 So taking the sum of all the angles in all the five triangles above,
 we get $5P + 360^\circ$.

Also, we can put the sum this way, too: $5 \cdot 180^\circ = 900^\circ$.

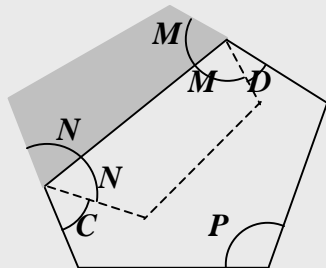
That's because the pentagon is made of five triangles, and the sum of three angles in each triangle is 180° .

So we get $5P + 360^\circ = 900^\circ \Rightarrow P = (900^\circ - 360^\circ)/5 = 540^\circ/5 = 108^\circ$.

Now, we have $N + N + C = 180^\circ$. So finding N , we can get C .
 How then can we find N ?

Getting back to the pentagon we get unfolding the figure with the angle C , we have this:

Fig. 1.4



$$D = 23^\circ.$$

Then, to begin with,
 the sum of all the angles in a tetragon is 360° ,
 since we can make a tetragon
 putting two triangles together.
 And next, every vertex angle is P ,
 since the pentagon is regular.

So looking at the tetragon in gray above, we get $N + M + 2P = 360^\circ$.

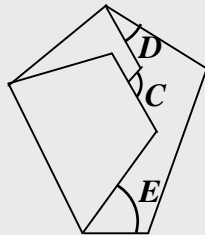
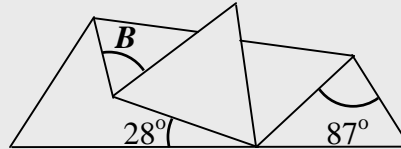
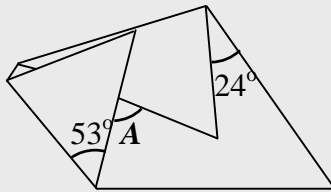
And we know $M + 2P = 78.5^\circ + 2 \cdot 108^\circ = 294.5^\circ$.

So we get $N = 360^\circ - (M + 2P) = 360^\circ - 294.5^\circ = 65.5^\circ$.

Now, we have $N + N + C = 180^\circ$. So we get $C = 180^\circ - 2N = 180^\circ - 131^\circ = 49^\circ$.

Suggestions or Solutions To the Problem 2

Assuming two regular (equilateral) triangles and a regular pentagon are folded the way below, find the three angles A , B , and C .

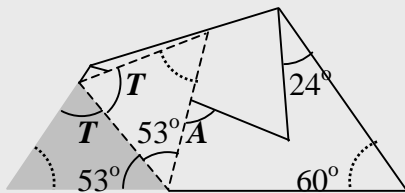


$$D = 23^\circ, \text{ and } E = 45^\circ.$$

Unfolding the figures above, we can unfold the way to the solution.

So first, unfolding one of the three figures the way below, we can see some angles that can help find the angle A .

Fig. 2.0



Then first, we can see two identical triangles.

One is in gray, and the other is made of dashed line segments.

Next, we know this:

Unfolding the figure completely, we get a regular triangle.

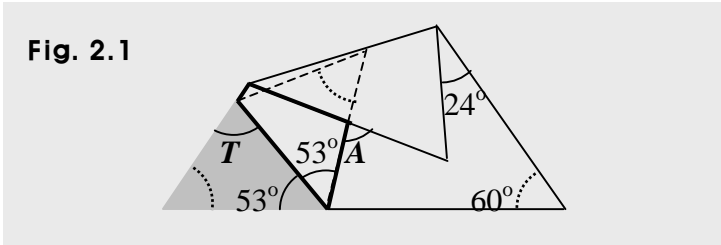
Each angle in a regular triangle is 60° .

And we know in any triangle, the three angles add up to 180° .

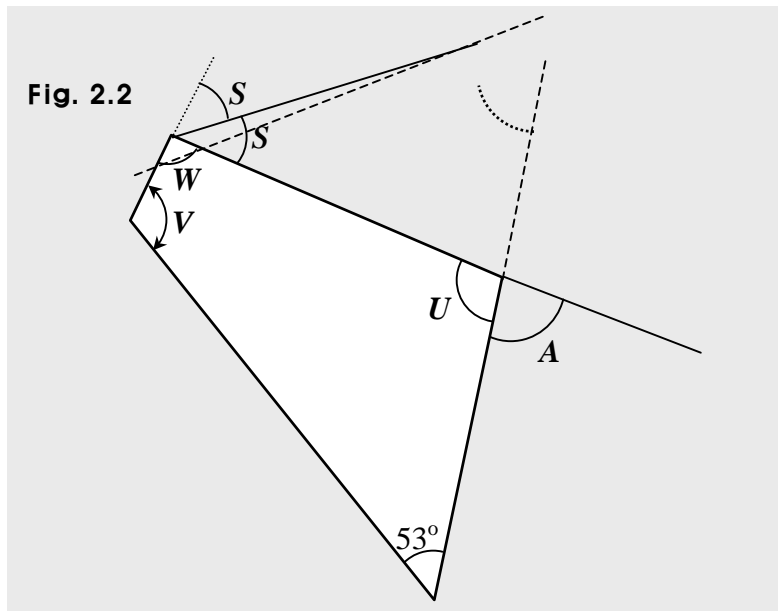
$$\text{So we get } T + 53^\circ + 60^\circ = 180^\circ \Rightarrow T = 180^\circ - 113^\circ = 67^\circ.$$

How then, can we get the angle A ?

Let's consider the tetragon indicated by thick line segments below.



And zooming down on the tetragon, we can put it the way below.

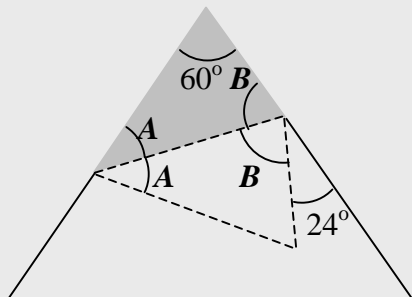


Then, assuming we can find the two angles V and W above, we can find the angle U . Then, we can get A , since U and A are supplement to each other, that is, $U + A = 180^\circ$.

To begin with, the angle S is the angle A in the problem 1 above, and $A = 42^\circ$. And we have $S + S + W = 180^\circ$. How?

In the problem 1, we unfolded one of the three figures the way below.

Fig. 2.3



And we found that $A = 42^\circ$.

So we get $S = 42^\circ \Rightarrow W = 180^\circ - 84^\circ = 96^\circ$.

And we have $V + T = 180^\circ$, and $T = 67^\circ$. So we get $V = 180^\circ - T = 180^\circ - 67^\circ = 113^\circ$.

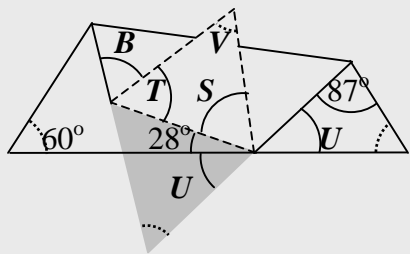
Now, we have $U + V + W + 53^\circ = 360^\circ$, and $U + A = 180^\circ$.

So first, we get $U = 360^\circ - V - W - 53^\circ = 360^\circ - 113^\circ - 96^\circ - 53^\circ = 360^\circ - 262^\circ = 98^\circ$.

Thus, next, we get $U + A = 180^\circ \Rightarrow A = 180^\circ - U = 180^\circ - 98^\circ = 82^\circ$.

Let's next, move on to the figure with the angle B , and unfold it the way below.

Fig. 2.4



Then first, we can see two identical triangles.

One is in gray, and the other is made of dashed line segments.

So next, we get $B + 2T = 180^\circ$. So finding T , we can get B .

How then can we find T ?

Fully unfolding the figure, we get a regular triangle, where every angle is 60° .

So we can see that $V = 60^\circ$, and thus, can get $Q + S + 60^\circ = 180^\circ$.
 So finding S , we can find Q . How then, can we find S ?

We can notice that $S = 28^\circ + U$, because unfolding the figure, we get two identical triangles, one is in gray, and the other is made of dashed line segments.
 So finding U , we can find S . Well then, how can we find U ?

We know that the original triangle is regular. So we get
 $U + 87^\circ + 60^\circ = 180^\circ \Rightarrow U = 180^\circ - 87^\circ - 60^\circ = 180 - 147^\circ = 33^\circ$.

Thus, we can now, get the angle B .

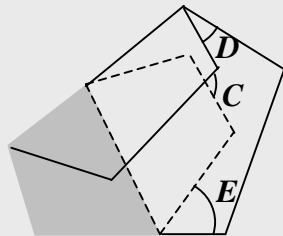
To begin with, we have $S = 28^\circ + U$, so we get $S = 28^\circ + 33^\circ = 61^\circ$.

So next, we get $Q + S + 60^\circ = 180^\circ \Rightarrow Q = 180^\circ - 61^\circ - 60^\circ = 180 - 121^\circ = 59^\circ$.

Thus, next, we get $B + 2T = 180^\circ \Rightarrow B = 180^\circ - 2T = 180^\circ - 118^\circ = 62^\circ$.

Let's next, move on to the figure with the angle C , and unfold it the way below.

Fig. 2.5

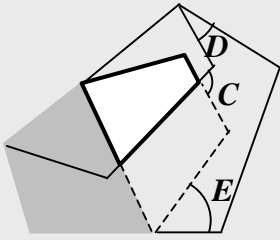


$$D = 23^\circ, \text{ and } E = 45^\circ.$$

Then first, we can see two identical tetragons.
 One is in gray, and the other is made of dashed line segments.

Let's consider the tetragon indicated by thick line segments below.

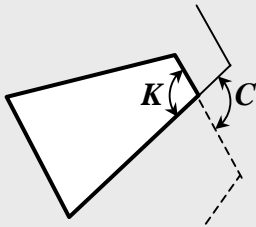
Fig. 2.6



$$D = 23^\circ, \text{ and } E = 45^\circ.$$

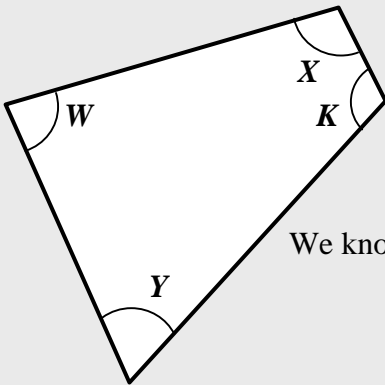
And magnifying the tetragon, we can put it the way below.

Fig. 2.7



So finding the angle K , we get the angle C , because $K = C$, since they are vertical angles. How then, can we find K ?

Fig. 2.8



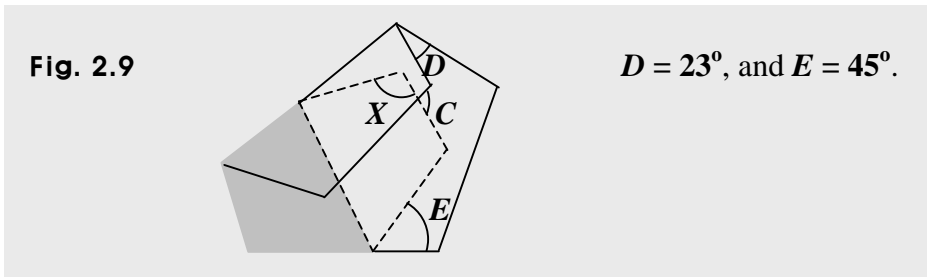
We know the four angles in a tetragon add up to 180° .

So we get $W + X + Y + K = 180^\circ$.

Thus, finding the three angles W , X , and Y ,
we can get the angle K .

How then can we find the three angles?

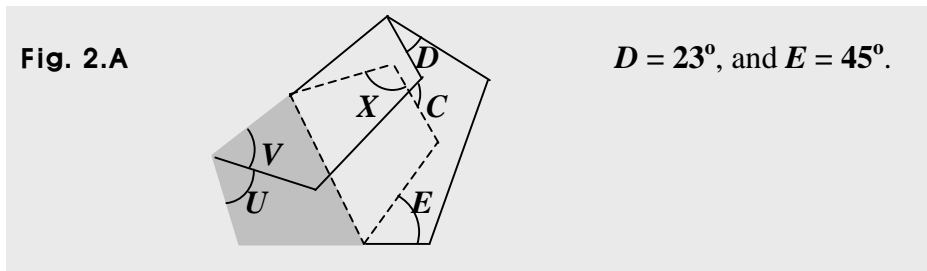
Let's first, get back to the figure where we can see the two *identical* tetragons.



Then, beginning with the angle X , what then can we say about the angle X ?

The angle X is the same as an angle in the tetragon in gray.
 What angle then is it?

Indicating two angles by U and V the way below, we can say that $X = U + V$.



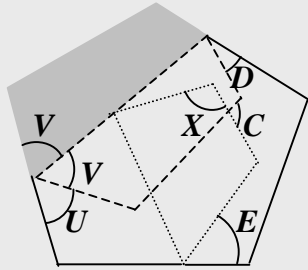
It's because the gray tetragon is the same as the tetragon made of dashed line segments. How then, can we find the two angles U and V ?

We have already found U , which is 49° , which is the solution to the problem 1.
 What then, about the angle V ?

We can set $U + 2V = 180^\circ$. How?

Unfolding the figure above, we can unfold the way to the solution.

Fig. 2.B



$D = 23^\circ$, and $E = 45^\circ$.

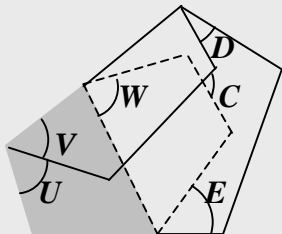
The tetragon in gray is identical to the tetragon made of dashed line segments.

So we get $U + 2V = 180^\circ \Rightarrow V = (180^\circ - U)/2 = (180^\circ - 49^\circ)/2 = 131^\circ/2 = 65.5^\circ$.

And we know $X = U + V$. So we get $X = 49^\circ + 65.5^\circ = 114.5^\circ$.

And let's next, move on to the angle W .

Fig. 2.C



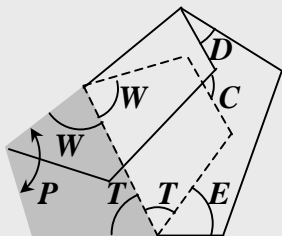
$D = 23^\circ$, and $E = 45^\circ$.

Then again, we can get the angle W using the fact that the gray tetragon is the same as the tetragon made of dashed line segments.

How can we use the fact though?

Using the fact, we can name some angles the way below.

Fig. 2.D



$D = 23^\circ$, and $E = 45^\circ$.

And we know that the sum of all the four angles in a tetragon is 360° , each of the five angles in a regular pentagon is 108° , and of course, the angle $P = U + V = 49^\circ + 65.5^\circ$.

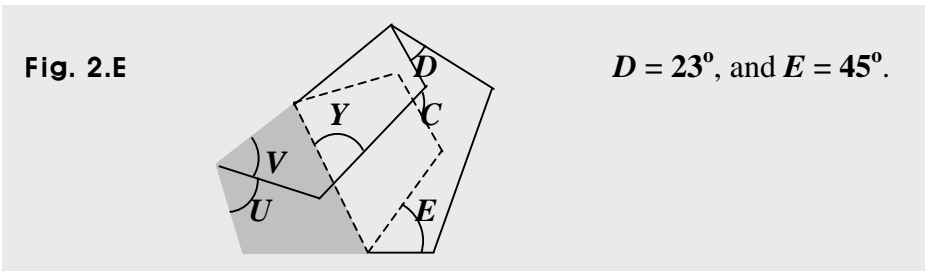
So we get $W + 114.5^\circ + 108^\circ + T = 360^\circ \Rightarrow W = 360^\circ - T - 222.5^\circ = 137.5^\circ - T$. Finding T thus, we can get W . How then can we find the angle T ?

We have $T + T + E = 180^\circ$, where $E = 45^\circ$.

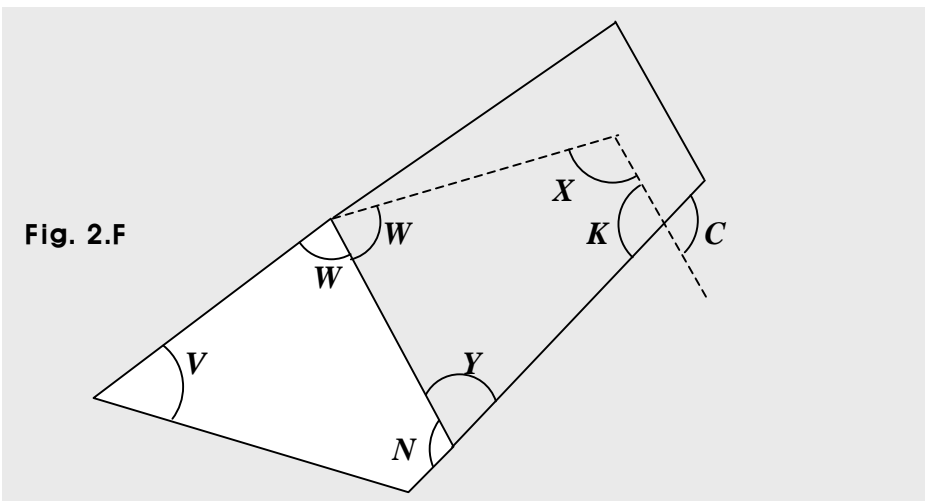
So we get $2T + 45^\circ = 180^\circ \Rightarrow T = (180^\circ - 45^\circ)/2 = 135^\circ/2 = 67.5^\circ$.

Thus, we get $W = 237.5^\circ - T = 137.5^\circ - 67.5^\circ = 70^\circ$.

And let's next, move on to the angle Y .



Magnifying the area where the angle Y is, we can get this:



Then, we can see that $N + Y = 180^\circ$, and also, that $V + W + N + 108^\circ = 360^\circ$.

That is, we have $Y = 180^\circ - N$, and $N = 360^\circ - 108^\circ - W - V = 252^\circ - W - V$.

Where is 108° from though?

Every angle in a regular pentagon is 108° , which is in this case next to the angle N . And we know $V = 65.5^\circ$, and $W = 70^\circ$, so we get $N = 252^\circ - 65.5^\circ - 70^\circ = 116.5^\circ$.

Thus, we get $Y = 180^\circ - N = 180^\circ - 116.5^\circ = 63.5^\circ$.

Now, we have $W + X + Y + K = 360^\circ$, and also, $K = C$ since K and C are vertical angles.

So we get $C = 360^\circ - (W + X + Y)$. And we have $W = 70^\circ$, $X = 114.5^\circ$, and $Y = 63.5^\circ$.

Thus, we get $C = 360^\circ - (70^\circ + 114.5^\circ + 63.5^\circ) = 360^\circ - 248^\circ = 118^\circ$.